

OPERATING REGIME OF AN EXPLOSIVE MAGNETIC FIELD  
 COMPRESSION GENERATOR WITH A CAPACITIVE LOAD  
 WITH A CONSIDERATION OF MAGNETIC FLUX LOSSES

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The operating regime of an explosive magnetic field compression generator with a capacitive load has been examined [1, 2] in the RLC circuit approximation. The case has been described [1] where the inductance  $L$  is a quadratic function of time and the resistance  $R$  (which determines all losses in the circuit) is linear during the operation of the generator. The situation has been investigated [2] where  $L$  is a linear function of time and  $R = \text{const}$ , or else  $L$  is an exponential function of time and it is assumed either that  $R = 0$  or that  $\alpha = R/L = \text{const}$  (which is characteristic for a series of explosive magnetic field compression generators [3-7]). For  $\alpha = \text{const}$ , a general solution was obtained for the change in flux in the RLC circuit.

Here we examine a more complex case which reliably describes the actual physical processes in a explosive magnetic field compression generator. These processes are characterized by a time-varying value of  $\alpha = \alpha(t)$ . We obtain 1) an asymptotic estimate of the oscillation process, which, in spite of the absence of an exact solution, can be used to determine the magnitude and variation of the frequency, the amplitude of the current and voltage in the final stages of the magnetic generator operation; and 2) exact solutions which consider the initial conditions under the assumption  $\alpha = \text{const}$  for an exponential and a linear time-dependence of the inductance. The solutions were investigated for a range of initial inductances 100-1000  $\mu\text{H}$  and a load capacity of  $10^{-10}$ - $10^{-8}$  F.

1. The operating regime of the explosive magnetic field compression generator is described within the framework of the circuit shown in Fig. 1. Here  $L_g$  and  $L_l$  are the inductances of the generator and the load,  $C$  is the capacitance of the load, and  $R$  is the effective resistance, which determines all losses in the circuit. The electrical current  $I$ , the magnetic flux  $\Phi = LI$ , and the voltage at the capacitor  $U = \frac{1}{C} \int_0^t I dt$  are described by the

equations

$$(LI)' + RI + \frac{1}{C} \int_0^t I dt = 0, \quad \Phi' + \frac{R}{L} \Phi + \frac{1}{C} \int_0^t \frac{\Phi}{L} dt = 0, \quad (1.1)$$

$$LU'' + (L' + R)U' + \frac{1}{C}U = 0$$

where  $L = L_g + L_l$  is the inductance of the circuit. It is assumed that at  $t = 0$  (at the start of the generator operation),  $\Phi = \Phi_0$ ,  $I = I_0$ , and  $U = 0$ . Let

$$L = L_0 f(\tau), \quad \tau = t/\tau_L, \quad (1.2)$$

where  $\tau_L$  is the characteristic time for changing the inductance;  $L_0$  is the initial inductance.

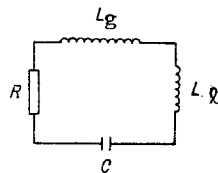


Fig. 1

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tance of the circuit;  $f(\tau) > 0$  is a monotonically decreasing differentiable function [ $f(0) = 1$ ,  $f'(\tau) < 0$ , and  $f''(\tau) > 0$ ; for  $\tau \rightarrow \infty$ ,  $f(\tau)$ ,  $f'(\tau)$ , and  $f''(\tau)$  tend to zero]. This dependence, for example, describes the change in the inductance of a helical generator in which the step of the helical windings increases along the length of the generator. We transform the integral-differential equations (1.1) into differential equations in a dimensionless form by using (1.2):

$$\begin{aligned} y_i'' + P_i(\tau)y_i' + Q_i(\tau)y_i &= 0, \\ y_1 = J = I/I_0, P_1 = 2f'/f + v, Q_1 = f''/f + v f'/f + v' + \theta^2/f, \\ y_2 = V = U\tau_L/(I_0L_0), P_2 = v + f'/f, Q_2 = \theta^2/f, \\ y_3 = \eta = \Phi/\Phi_0, P_3 = v, Q_3 = v' + \theta^2/f, v(\tau) = \alpha(\tau)\tau_L, \\ \alpha(\tau) = R(\tau)/L(\tau), \theta^2 = \tau_L^2/(CL_0) = \text{const} \quad (i = 1, 2, 3). \end{aligned} \quad (1.3)$$

Hereafter the subscript 1 denotes quantities characterizing the current, 2 the voltage, and 3 the magnetic flux. From Eqs. (1.1)-(1.3), it follows that

$$\eta = Jf, V' = \theta^2J. \quad (1.4)$$

We write the initial conditions for  $\tau = 0$ , by using Eqs. (1.1):

$$J = 1, J' + f'/f + v = 0, V = 0, V' - \theta^2 = 0, \eta = 1, \eta' + v = 0. \quad (1.5)$$

If we reduce (1.3) to two terms by using the formulation

$$\begin{aligned} y_i &= \exp\left[-\frac{1}{2}\int_0^\tau P_i(\tau) d\tau\right] x_i(\tau), \\ y_1 &= \frac{1}{f(\tau)} A(\tau) x_1(\tau), \quad y_2 = \sqrt{\frac{1}{f(\tau)}} A(\tau) x_2(\tau), \\ y_3 &= A(\tau) x_3(\tau), \quad A(\tau) = \exp\left[-\frac{1}{2}\int_0^\tau v(\tau) d\tau\right], \end{aligned} \quad (1.6)$$

we have

$$\begin{aligned} x_i'' + q_i(\tau)x_i &= 0, \quad q_i = Q_i - P_i^2/4 - P_i'/2, \\ q_1 &= q_3 = \theta^2/f + v'/2 - v^2/4, \\ q_2 &= \frac{\theta^2}{f} + \frac{1}{4}\left(\frac{f'}{f}\right)^2 - \frac{1}{4}v^2 - \frac{v'}{2} - \frac{1}{2}\frac{f''}{f} - \frac{vf'}{2f}. \end{aligned} \quad (1.7)$$

By using the properties of the solutions to second-order equations of the type (1.7) and the asymptotic formulas (the WKB approximation [8]), the character of the process can be determined from features of the reduced coefficients  $q_i(\tau)$ . Let the function  $v(\tau)$  (which characterizes the magnetic flux losses) and its derivatives be bounded in time; then from (1.7) and the properties of the function  $f(\tau)$  (1.2), we obtain that  $q_i(\tau)$  remains positive during generator operation, independent of the sign of  $q_i(0)$  at the initial moment of time. From this it follows that if the capacitance  $C$  is large ( $\theta^2 \ll 1$ ) and if  $q_i(0) < 0$  at the beginning of the process, there is an aperiodic regime, which becomes oscillatory for  $q_i(\tau) > 0$ . For small values of the capacitance  $C$  ( $\theta^2 \gg 1$ ) and for  $q_i(0) > 0$ , oscillations occur over the whole operation of the generator.

If  $v'''(\tau)$  is continuous for  $\tau \geq 0$  [so that  $q_i''(\tau)$  is also continuous] and if the condition

$$\begin{aligned} q_i(\tau) > 0, \quad \int_0^\infty |\alpha_i(\tau)| d\tau < \infty, \\ \alpha_i(\tau) = \frac{1}{8} \frac{q_i''(\tau)}{q_i^{3/2}(\tau)} - \frac{5}{32} \frac{(q_i'(\tau))^2}{q_i^{5/2}(\tau)} \quad (i = 1, 2, 3), \end{aligned} \quad (1.8)$$

is fulfilled [8], then Eq. (1.7) has two asymptotic WKB solutions for  $\tau \rightarrow \infty$ :

$$\begin{aligned} x_{i1} &= q_i^{-1/4} \cos \int_0^\tau \sqrt{q_i(\tau)} d\tau + \varepsilon_{i1}(\tau), \quad x_{i2} = \\ &= q_i^{-1/4} \sin \int_0^\tau \sqrt{q_i(\tau)} d\tau + \varepsilon_{i2}(\tau), \end{aligned} \quad (1.9)$$

$$\varepsilon_{ij}(\tau) \leq B \int_{\tau}^{\infty} |\alpha_i(\tau)| d\tau, \quad \lim_{\tau \rightarrow \infty} \varepsilon_{ij}(\tau) = 0$$

$$(B = \text{const}, i = 1, 2, 3, \quad j = 1, 2).$$

If in addition to (1.8), the condition

$$\lim_{\tau \rightarrow \infty} \frac{q_i'}{q_i^{3/2}} = 0, \quad (1.10)$$

is fulfilled, the solutions (1.9) are linearly independent and can be differentiated [8]. Then for  $\tau \rightarrow \infty$ , we obtain

$$x'_{i1} = -q_i^{1/4} \sin \int_0^{\tau} \sqrt{q_i} d\tau, \quad x'_{i2} = q_i^{1/4} \cos \int_0^{\tau} \sqrt{q_i} d\tau. \quad (1.11)$$

By using Eqs. (1.6), (1.9), and (1.11), we can estimate the frequency and amplitude of the oscillating process. When  $\theta^2 \gg 1$ , then  $q_i \approx \theta^2/f > 0$  according to (1.7), which is characteristic for a helical inductance, which is distributed exponentially along the length of the generator or is not too far from it in any case; for example, for [8]

$$f = \exp \left[ - \sum_j a_j \tau^{b_j} \right], \quad a_j = \text{const}, \quad b_j = \text{const} \quad (1.12)$$

( $j$  is any positive integer), or for [3]

$$f = \exp [h(1 - \exp(v\tau)) - v\tau], \quad h = \text{const}, \quad v = \text{const}. \quad (1.13)$$

Conditions (1.8) and (1.10) are fulfilled for  $q_i = \theta^2/f$ , where  $f$  is determined by Eq. (1.12) or (1.13). Then Eqs. (1.6) and (1.7) are simplified, and the asymptotic solution to Eqs. (1.3) has the form

$$J = \theta^{-1/2} \frac{A(\tau)}{f(\tau)^{3/4}} D_1(\tau), \quad V = \theta^{-1/2} \frac{A(\tau)}{f(\tau)^{1/4}} D_2(\tau), \quad (1.14)$$

$$D_i = C_{i1} \cos B(\tau) + C_{i2} \sin B(\tau),$$

$$B(\tau) = \theta \int_0^{\tau} f(\tau)^{-1/2} d\tau, \quad C_{ij} = \text{const} \quad (i, j = 1, 2).$$

Because  $v(\tau)$  is a bounded function, it follows from (1.6) that  $A(\tau)$  decreases as  $\tau$  increases. Therefore the amplitude of the voltage in (1.14) increases, if the function

$$f(\tau) = o[A(\tau)^4] = o \left[ \exp \left( -2 \int_0^{\tau} v(\tau) d\tau \right) \right],$$

decreases, and the current amplitude increases, if

$$f(\tau) = o[A(\tau)^{4/3}] = o \left[ \exp \left( -\frac{2}{3} \int_0^{\tau} v(\tau) d\tau \right) \right].$$

Here  $F(\tau) = o[g(\tau)]$  means that, for  $\tau \gg 1$ ,  $F(\tau)$  is an infinitely small quantity of high order in comparison to  $g(\tau)$ .

We now estimate the amplitude  $U^* = LI'$  (in a the dimensionless form  $fJ'$ ), which characterizes the voltage between the cone of the central expansion tube and the windings of the helix [3], for  $q_i = \theta^2/f$ . From (1.1)-(1.4) we obtain

$$fJ' = -(V + Jf' + v\eta). \quad (1.15)$$

By substituting the solution (1.14) into (1.15) we find

$$fJ' = -\frac{\theta^{-1/2} A(\tau)}{f(\tau)^{1/4}} \left[ D_2(\tau) + \frac{f'(\tau)}{\sqrt{f(\tau)}} D_1(\tau) + v \sqrt{f(\tau)} D_1(\tau) \right]. \quad (1.16)$$

The condition (1.10) for  $q_i = \theta^2/f$  has the form

$$\lim_{\tau \rightarrow \infty} \frac{q_i'(\tau)}{q_i^{3/2}(\tau)} = \lim_{\tau \rightarrow \infty} \frac{f'(\tau)}{\sqrt{f(\tau)}}. \quad (1.17)$$

By using Eqs. (1.17), and also the properties of the function  $f(\tau)$  from (1.16), we obtain that  $Jf' \ll V$  and  $v\eta \ll V$  as  $\tau$  increases. Consequently, for  $t/\tau_L \gg 1$ , the amplitude  $U^*$  can be estimated from the value of the voltage amplitude at the capacitor.

In order to estimate the oscillation frequency  $\omega$  (the carrier frequency, because in a real system a spectrum of frequencies is observed), we use the formula

$$B(\tau + T) - B(\tau) = \theta \left[ \int_0^{\tau+T} \frac{d\tau}{Vf(\tau)} - \int_0^{\tau} \frac{d\tau}{Vf(\tau)} \right] = 2\pi \quad (1.18)$$

in the solutions (1.14), where  $B'(\tau) = \theta f(\tau)^{-1/2}$  and where  $T = 2\pi/(\omega\tau_L)$  is the oscillation period. By expanding the left side of (1.17) in a Taylor series (assuming that  $T \rightarrow 0$  for  $\tau \rightarrow \infty$ ) and keeping only the linear term, we find

$$\omega = \frac{\theta}{\tau_L} f(\tau)^{-1/2} = \frac{1}{\sqrt{L(\tau)C}}, \quad L(\tau) = L_0 f(\tau), \quad \tau = \frac{t}{\tau_L}. \quad (1.19)$$

Thus, we obtain that for  $\theta^2 \gg 1$  (and for small values of the capacitance  $C$ ), the oscillation frequency increases with time and does not depend on the magnetic flux losses, but depends on the capacitance of the condenser and the induction  $L(\tau)$  (1.19). We now examine the case where it is possible to obtain exact solutions to Eqs. (1.1) and (1.3).

2. Let the variation of the inductance of the explosive magnetic field compression generator be exponential during its operation and be described in the models [2, 4-7]:

$$f(\tau) = \exp(-\tau), \quad \tau = t/\tau_L. \quad (2.1)$$

Here  $\tau_L$  is operating time over which the inductance of the generator varies by a factor of  $e$ . Let

$$\alpha = R(\tau)/L(\tau) = \text{const} \quad (v = \text{const}) \quad (2.2)$$

over the whole time of the generator operation [3]. Then we write the coefficients in Eqs. (1.3) and (1.7) as

$$P_1 = v - 2, \quad Q_1 = 1 - v + \theta^2 \exp(\tau), \quad q_1 = \theta^2 \exp(\tau) - v^2/4, \\ P_2 = v - 1, \quad Q_2 = \theta^2 \exp(\tau), \quad q_2 = \theta^2 \exp(\tau) - (v - 1)^2/4.$$

After we represent the general solution to Eqs. (1.3) in the form of cylinder functions [9] and solve the initial-value problem, we have from the properties of the Bessel functions [10] that

$$J = \pi\theta \exp\left(\frac{2-v}{2}\tau\right) [Y_{v-1}(2\theta)J_v(\delta) - J_{v-1}(2\theta)Y_v(\delta)], \quad (2.3) \\ V = \pi\theta \exp\left(\frac{1-v}{2}\tau\right) [J_{v-1}(2\theta)Y_{v-1}(\delta) - Y_{v-1}(2\theta)J_{v-1}(\delta)], \quad \delta = 2\theta \exp(\tau/2).$$

If there are no magnetic flux losses ( $R = v = 0$ ), we obtain a solution from (2.3) which coincides with that presented in [2]:

$$J = \pi\theta \exp(\tau) [J_1(2\theta)Y_0(\delta) - Y_1(2\theta)J_0(\delta)], \\ V = \pi\theta^2 \exp(\tau/2) [J_1(2\theta)Y_1(\delta) - Y_1(2\theta)J_1(\delta)].$$

Oscillations are observed at the end of the generator operation ( $\tau \rightarrow \infty$ ). From (2.3) it follows that

$$J = \sqrt{\pi\theta} \exp\left(\frac{3-2v}{4}\tau\right) [Y_{v-1}(2\theta) \cos \beta - J_{v-1}(2\theta) \sin \beta], \quad (2.4) \\ V = \sqrt{\pi\theta^3} \exp\left(\frac{1-2v}{4}\tau\right) [J_{v-1}(2\theta) \cos \beta + Y_{v-1}(2\theta) \sin \beta], \quad \beta = \\ = 2\theta \exp(\tau/2) - v\pi/2 - \pi/4.$$

For  $\theta \rightarrow \infty$ , we find from (2.3) that

$$J = \exp\left(\frac{3-2v}{4}\tau\right) \cos \mu, \quad V = \theta \exp\left(\frac{1-2v}{4}\tau\right) \sin \mu, \quad \mu = 2\theta [\exp(\tau/2) - 1]. \quad (2.5)$$

In this case we can apply the asymptotic approximation (1.14) in the form

$$J = \theta^{-1/2} \exp\left(\frac{3-2\nu}{4}\tau\right) [C_{11} \cos \mu + C_{12} \sin \mu], \quad (2.6)$$

$$V = \theta^{-1/2} \exp\left(\frac{1-2\nu}{4}\tau\right) [C_{21} \cos \mu + C_{22} \sin \mu], \quad C_{ij} = \text{const} \quad (i, j = 1, 2).$$

By comparing (2.5) and (2.6), we see that the periods of these oscillations are equal, and the amplitudes of the corresponding quantities are mutually proportional. The amplitude increases with time (2.4)-(2.6) for  $J$  if  $\nu < 3/2$ , and for  $V$  if  $\nu < 1/2$ .

Thus, when  $\alpha = R/L = \text{const}$ , the dimensionless values of the magnetic flux, the current, and the voltage depend on the parameters  $\nu$  and  $\theta$  during the operation of the explosive magnetic field compression generator. Based on data [3-7], we take the following range of the parameter variations:

$$\begin{aligned} \tau_L &= 1 \dots 20 \text{ } \mu\text{sec} \quad L_0 = 100 \dots 1000 \text{ } \mu\text{H} \\ C &= 10^{-10} \dots 10^{-8} \text{ F}, \quad \nu = 0,1 \dots 0,4. \end{aligned} \quad (2.7)$$

This combination of dimensional parameters corresponds to  $\theta = 1-200$ . Calculations showed that for  $1 \leq \theta \leq 2.5$ , the amplitude envelope of the oscillations can be described by Eqs. (2.5) with an absolute error  $\leq 5\%$ . The voltage at the capacitor and the current in the circuit and the end of the generator operation can be estimated from the envelope of the oscillation amplitudes. The current amplitude  $J$  is independent of  $\theta$ , and therefore of the capacitance  $C$ . The voltage amplitude is proportional to  $I_0 \sqrt{L_0/C}$ . The effect of flux losses on the oscillation amplitude is very substantial.

We note that in the case of a purely inductive load in the circuit for  $C \rightarrow \infty$  ( $\theta \rightarrow 0$ ), we have from the solution to (2.3)

$$J = \exp [(1 - \nu)\tau]. \quad (2.8)$$

Comparison of (2.5) and (2.8) shows that the growth in the current amplitude during the generator operation with a capacitive load is more intensive than for operation with an inductive load for  $\nu < 1/2$ .

If we assume that the energy amplification coefficient  $K_E = (1/2)CU^2 / [(1/2)L_0 I_0^2]$  is the ratio of the condenser energy to the energy of the initial powering of the generator, we obtain from (1.3) that  $K_E = V^2/\theta^2$ . The value of  $K_E$  can be estimated by using (2.5).

Figure 2 shows the logarithm of the voltage amplitude in (2.5) as a function of time  $\tau$  for various values of  $\nu$ :

$$\ln \frac{V}{\theta} = \ln \frac{U}{I_0 \sqrt{L_0/C}} = \frac{1}{2} \ln K_E = \frac{1-2\nu}{4} \tau.$$

By estimating the oscillation frequency from Eq. (1.18), we find from (2.4) or (2.5) that the frequency  $\omega$  is independent of the magnetic flux losses and has the form

$$\omega = \frac{1}{\tau_L} = \frac{\pi}{\ln [\pi \exp(-\tau/2) \theta^{-1} + 1]}. \quad (2.9)$$

As  $\theta$  or  $\tau$  increases, the oscillation frequency increases and its dependence becomes analogous to (1.19)

$$\omega = \frac{\theta}{\tau_L} \exp(\tau/2) = \frac{1}{\sqrt{L(\tau)C}}, \quad L(\tau) = L_0 \exp(-\tau), \quad \tau = \frac{t}{\tau_L}. \quad (2.10)$$

Figure 3 shows the dimensionless frequency  $\omega \tau_L$  as a function of  $\theta \cdot \exp(\tau/2)$  according to Eqs. (2.9) and (2.10) (solid and dashed lines). Estimates showed that the absolute error in the calculation of  $\omega$  according to Eq. (2.10) does not exceed 10% for  $\theta \cdot \exp(\tau/2) \sim 16$ .

3. We now examine a coaxial type of explosive magnetic field compression generator (or a helical generator whose coils are wound with a constant step). Here the behavior of the inductance is written in the form [2, 11]

$$f = 1 - \tau, \quad (3.1)$$

where  $\tau = t/\tau_L^*$  and  $\tau_L^*$  is the characteristic time over which the inductance decreases to zero. Then the coefficients for Eqs. (1.3) and (1.7) are as follows:

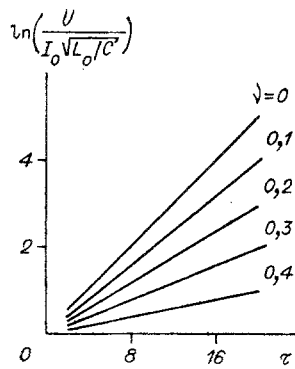


Fig. 2

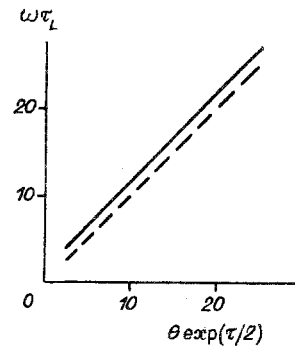


Fig. 3

$$P_1 = \nu - \frac{2}{1-\tau}, \quad Q_1 = \frac{\theta^2}{1-\tau} - \frac{\nu}{1-\tau} + \nu', \quad q_1 = \frac{\theta^2}{1-\tau} + \frac{\nu'}{2} - \frac{\nu^2}{4},$$

$$P_2 = \nu - \frac{1}{1-\tau}, \quad Q_2 = \frac{\theta^2}{1-\tau}, \quad q_2 = \frac{\theta^2}{1-\tau} + \frac{1}{4(1-\tau)^2} + \frac{\nu}{2(1-\tau)} - \frac{\nu^2}{4} - \frac{\nu'}{2}.$$

In accordance with the determination of  $\tau$  in (3.1), the explosive magnetic field compression generator operates for  $0 < \tau < 1$ . The WKB asymptotic approximation cannot be used, since for  $\tau \rightarrow 1$  conditions analogous to (1.8) [8] are not fulfilled. For the case (2.2), when  $R(\tau)/L(\tau) = \alpha = \text{const}$  ( $\nu = \text{const}$ ), the solution to Eqs. (1.3) with the initial conditions (1.5) can be represented in the form of degenerate hypergeometric functions [9]. We obtain in the notations of [10] that

$$J = \nu \exp(-\nu) \Gamma\left(1 - \frac{\theta^2}{\nu}\right) \left\{ U\left[-\frac{\theta^2}{\nu}, 1, \nu\right] M\left[1 - \frac{\theta^2}{\nu}, 2, (1-\tau)\nu\right] + \right. \quad (3.2)$$

$$\left. + M\left[-\frac{\theta^2}{\nu}, 1, \nu\right] U\left[1 - \frac{\theta^2}{\nu}, 2, (1-\tau)\nu\right] \right\},$$

$$V = \theta^2 \exp(-\nu) \Gamma\left(-\frac{\theta^2}{\nu}\right) \left\{ M\left[-\frac{\theta^2}{\nu}, 1, \nu\right] U\left[-\frac{\theta^2}{\nu}, 1, (1-\tau)\nu\right] + \right.$$

$$\left. + U\left[-\frac{\theta^2}{\nu}, 1, \nu\right] M\left[-\frac{\theta^2}{\nu}, 1, (1-\tau)\nu\right] \right\}$$

where  $\Gamma(x)$  is the gamma function. When the magnetic flux losses are small ( $\nu \rightarrow 0$ ,  $\theta^2/\nu \rightarrow \infty$ , and  $\theta$  is fixed), we obtain from (3.2) a solution [10], which is represented in the form of cylinder functions and which coincides with [2]:

$$J = \frac{\pi\theta}{\sqrt{1-\tau}} [Y_0(2\theta)J_1(2\theta\sqrt{1-\tau}) - J_0(2\theta)Y_1(2\theta\sqrt{1-\tau})],$$

$$V = \pi\theta^2 [Y_0(2\theta)J_0(2\theta\sqrt{1-\tau}) - J_0(2\theta)Y_0(2\theta\sqrt{1-\tau})].$$

At the end of the generator operation for  $\tau \rightarrow 1$ , the solution (3.2) is not expressed in terms of trigonometric functions, as opposed to the case derived in Sec. 2. For small values of the capacitance ( $C \rightarrow 0$ ,  $\theta^2/\nu \rightarrow \infty$ , and  $\nu$  fixed) the solution (3.2) is simplified, and oscillations are observed:

$$J = \frac{\exp(-\nu\tau/2)}{(1-\tau)^{3/4}} \cos[2\theta(1-\sqrt{1-\tau})], \quad (3.3)$$

$$V = \frac{\theta \exp(-\nu\tau/2)}{(1-\tau)^{1/4}} \sin[2\theta(1-\sqrt{1-\tau})].$$

The oscillation frequency  $\omega$  in the solution (3.3) can be estimated from formulas analogous to (1.18) and (1.19):

$$\omega = 1/\sqrt{L(\tau)C}, \quad L(\tau) = L_0(1-\tau), \quad \tau = t/\tau_L^*.$$

It follows from (3.3) that the current amplitude decreases for  $\tau \leq 1 - 1/(2\nu)$ , but increases for  $\tau > 1 - 1/(2\nu)$ . The voltage amplitude has a similar dependence for  $\tau \leq 1 - 3/(2\nu)$ . The oscillation frequency increases with time.

We note that the characteristic time  $\tau_L^*$ , determined from (3.1), is much larger than  $\tau_L$  from (2.1). Therefore, if the values of  $\alpha$  and the initial inductance and capacitance are identical, the dimensionless parameters  $\nu$  and  $\theta$  at a given point, which are proportional to

$\tau_L^*$  according to (1.3), are much higher than the corresponding parameters in Sec. 2. Calculations showed that the growth in the oscillation frequency and amplitude in Eq. (3.3) is less intense than for helices with an exponential inductance law.

This investigation makes it possible to estimate the basic parameters, which characterize the operation of an explosive magnetic field compression generator with a capacitive load.

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